FINAL ORDINARIO MATEMÁTICAS I

31/05/2018

1 a) $3^{x-1} = 9^{\sqrt{x}}$

$3^{x-1} = (3^2)^{\sqrt{x}}$

$3^{x-1} = 3^{2\sqrt{x}} \longrightarrow x-1 = 2\sqrt{x}$

$(x-1)^2 = (2\sqrt{x})^2$

$x^2+1-2x = 4x$

$x^2-6x+1=0 \longrightarrow x = \dfrac{6 \pm \sqrt{6^2-4}}{2}$

$x = \dfrac{6 \pm \sqrt{32}}{2}$

$x = \dfrac{6 \pm 4\sqrt{2}}{2}$

$\boxed{x = 3 \pm 2\sqrt{2}}$

b) $\log 4 + 2 \cdot \log(x-3) = \log x$

$\log 4 + \log(x-3)^2 = \log x$

$\log 4 \cdot (x-3)^2 = \log x \longrightarrow 4(x-3)^2 = x$

$4(x^2+9-6x) = x$

$4x^2+36-24x = x$

$4x^2-25x+36=0$

$x = \dfrac{25 \pm \sqrt{25^2-4 \cdot 4 \cdot 36}}{2 \cdot 4} = \dfrac{25 \pm \sqrt{625-576}}{8} = \dfrac{25 \pm \sqrt{49}}{8}$

$x = \dfrac{25 \pm 7}{8}$

$\boxed{x_1 = 4 \quad x_2 = \dfrac{9}{4}}$

2) GAUSS

$$\left.\begin{array}{r} x+y+z=2 \quad ① \\ 2x+3y+5z=11 \quad ② \\ x-5y+6z=29 \quad ③ \end{array}\right] \begin{array}{c} \longrightarrow \\ \xrightarrow{②-2\cdot①} \\ \xrightarrow{③-①} \end{array} \left.\begin{array}{r} x+y+z=2 \quad ① \\ y+3z=7 \quad ② \\ -6y+5z=27 \quad ③ \end{array}\right]$$

$$\begin{array}{c} \longrightarrow \\ \longrightarrow \\ \xrightarrow{③+6②} \end{array} \left.\begin{array}{r} x+y+z=2 \quad ① \\ y+3z=7 \quad ② \\ 23z=69 \quad ③ \end{array}\right] \longrightarrow z=\frac{69}{23} \longrightarrow \boxed{z=3}$$

② $y+3z=7$

$y+3\cdot 3=7 \longrightarrow y=7-9 \longrightarrow \boxed{y=-2}$

① $x+\cancel{q}y+z=2$

$x-2+3=2 \longrightarrow x=2+2-3 \longrightarrow \boxed{x=1}$

3)

$\cos 2x - \operatorname{sen}^2 x = 1 \longleftarrow \cos 2x = \cos^2 x - \operatorname{sen}^2 x$

$\cos^2 x - \operatorname{sen}^2 x - \operatorname{sen}^2 x = 1 \longleftarrow \cos^2 x = 1 - \operatorname{sen}^2 x$

$\cancel{1} - \operatorname{sen}^2 x - 2\operatorname{sen}^2 x = \cancel{1}$

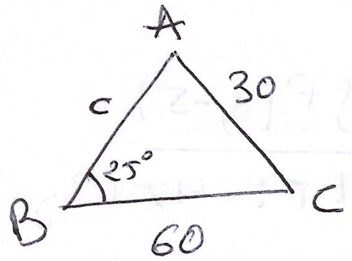
$-3\operatorname{sen}^2 x = 0 \longrightarrow \operatorname{sen} x = 0$

$x = \operatorname{arc\,sen} 0 = 0 + 180K$

$K=0,1,2$

$$\boxed{\begin{array}{l} x = 0 + 180\cdot K \\ \quad K=0,1,2\ldots \end{array}}$$

4)



$\hat{B} = 25°$ $b = 30$ cm $a = 60$ cm

$$\frac{a}{\operatorname{sen}\hat{A}} = \frac{b}{\operatorname{sen}\hat{B}} \rightarrow \frac{60}{\operatorname{sen}\hat{A}} = \frac{30}{\operatorname{sen}25} \rightarrow \operatorname{sen}\hat{A} = \frac{60}{30}\operatorname{sen}25$$

$$\operatorname{sen}\hat{A} = 2\cdot\operatorname{sen}25 = 2\cdot 0{,}42 = 0{,}84 \rightarrow \boxed{\hat{A} = 57{,}14°}$$

$$\hat{A}+\hat{B}+\hat{C} = 180 \rightarrow \hat{C} = 180-25-57{,}14 \rightarrow \boxed{\hat{C} = 97{,}86°}$$

$$\frac{c}{\operatorname{sen}\hat{C}} = \frac{b}{\operatorname{sen}\hat{B}} \rightarrow \frac{c}{\operatorname{sen}98} = \frac{30}{\operatorname{sen}25} \rightarrow c = 30\cdot\frac{\operatorname{sen}98}{\operatorname{sen}25}$$

$$\boxed{c = 70{,}71 \text{ cm}}$$

5) $\vec{u}(4,-3)$

$\vec{v}(9,m)$

a) $\vec{u} \perp \vec{v} \rightarrow \vec{u}\cdot\vec{v} = 0$

$\vec{u}\cdot\vec{v} = 4\cdot 9 - 3\cdot m$

$4\cdot 9 - 3\cdot m = 0$

$36 - 3m = 0$

$\boxed{m = 12}$

b) $|\vec{u}| = |\vec{v}|$

$$\sqrt{4^2+(-3)^2} = \sqrt{9^2+m^2} \rightarrow 4^2+(-3)^2 = 9^2+m^2$$

$16+9 = 81+m^2$

$m^2 = 16+9-81$

$m^2 = -56$

$m = \sqrt{-56} = \sqrt{(-1)\cdot 56}$

$\boxed{m = \pm i\sqrt{56}}$

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$$d(P,r) = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2+B^2}} \qquad \begin{cases} P(3,-5) \\ r:\ 4x+3y-2=0 \end{cases}$$

$$d(P,r) = \frac{|4\cdot 3 + 3(-5) - 2|}{\sqrt{4^2+3^2}} = \frac{|12-15-2|}{\sqrt{16+9}} = \frac{5}{\sqrt{25}} = 1$$

Ecuación de la circunferencia:

$$(x-3)^2 + (y+5)^2 = 1^2$$

$$x^2+9-6x+y^2+25+10y-1=0$$

$$\boxed{x^2+y^2-6x+10y+33=0}$$

7

$$f(x) = \begin{cases} x+1 & x \le 1 \\ 4-ax^2 & x > 1 \end{cases}$$

Condiciones continuidad:

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$$

$$f(1) = 2$$

$$\left.\begin{array}{l} \lim\limits_{x \to 1^+} 4-ax^2 = 4-a \\ \lim\limits_{x \to 1^-} x+1 = 2 \end{array}\right\} \begin{array}{l} 4-a=2 \\ a=4-2 \\ \boxed{a=2} \end{array}$$

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a) $$\lim_{x \to \infty} \frac{\sqrt{x^2+1} - 1}{x^2} = \lim_{x \to \infty} \left(\frac{\sqrt{x^2+1}}{x^2} - \frac{1}{x^2} \right) =$$

$$= \lim_{x \to \infty} \left(\sqrt{\frac{x^2+1}{x^4}} - \frac{1}{x^2} \right) = \lim_{x \to \infty} \left(\sqrt{\frac{x^2}{x^4} + \frac{1}{x^4}} - \frac{1}{x^2} \right) =$$

$$= \lim_{x \to \infty} \left(\sqrt{\frac{1}{x^2} + \frac{1}{x^4}} - \frac{1}{x^2} \right) = \sqrt{0+0} - 0 = 0$$

b) $$\lim_{x \to 2} \frac{x-2}{x^2-5x+6} = \lim_{x \to 2} \frac{\cancel{(x-2)}}{\cancel{(x-2)}(x-3)} =$$

$$= \lim_{x \to 2} \frac{1}{x-3} = \frac{1}{2-3} = \frac{1}{-1} = -1$$

9

a) $$y = 2x \cdot \sqrt{2x-1} = 2 \cdot \sqrt{2x^3 - x^2}$$

$$y' = \cancel{2} \cdot \frac{6x^2 - 2x}{\cancel{2}\sqrt{2x^3 - x^2}} = \frac{2x(3x^2-2)}{x\sqrt{2x-1}} = 2\,\frac{3x^2-2}{\sqrt{2x-1}}$$

b) $$y = tg\,\frac{x+1}{x-1}$$

$$y' = \frac{1}{1 + tg^2\,\frac{x+1}{x-1}} \cdot \frac{(x-1)-(x+1)}{(x-1)^2} =$$

$$y' = \frac{1}{1 + tg^2\,\frac{x+1}{x-1}} \cdot \frac{(-2)}{(x-1)^2}$$

10

$$f(x) = \frac{2x^2}{x-1} \qquad \text{Dom: } \mathbb{R} - \{1\}$$

a) Asíntota vertical en $\boxed{x = 1}$, posición:

$$\lim_{x \to 1^-} \frac{2x^2}{x-1} \approx \frac{2 \cdot 0{,}99^2}{0{,}99 - 1} \left(\frac{+}{-}\right) = -\infty$$

$$\lim_{x \to 1^+} \frac{2x^2}{x-1} \approx \frac{2 \cdot 1{,}01^2}{1{,}01 - 1} \left(\frac{+}{+}\right) = +\infty$$

Estudio:

$$\lim_{x \to \pm\infty} \frac{2x^2}{x-1} = \lim_{x \to \pm\infty} \frac{2x^2 + 2 - 2}{x-1} =$$

$$= \lim_{x \to \pm\infty} \left(\frac{2x^2 - 2}{x-1} + \frac{2}{x-1} \right) =$$

$$= \lim_{x \to \pm\infty} \left(2\frac{x^2 - 1}{x-1} + \frac{2}{x-1} \right) =$$

$$= \lim_{x \to \pm\infty} \left(2\frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} + \frac{2}{x-1} \right) =$$

$$= \lim_{x \to \pm\infty} \left(2(x+1) + \frac{2}{x-1} \right) \approx \lim_{x \to \pm\infty} 2(x+1)$$

Asíntota oblicua: $\boxed{y = 2x + 2}$

Posición relativa $\left(+ \frac{2}{x-1} \right)$:

b) $f'(x) = \dfrac{4x(x-1) - 2x^2}{(x-1)^2} = \dfrac{4x^2 - 4x - 2x^2}{(x-1)^2}$

$f'(x) = \dfrac{2x^2 - 4x}{(x-1)^2}$

Puntos singulares: $f'(x) = 0 \rightarrow 2x^2 - 4x = 0$

$2x(x-2) = 0$

$\begin{cases} x_1 = 0 \\ x_2 = 2 \end{cases}$

$\begin{cases} x_1 = 0 \rightarrow f(0) = 0 \rightarrow (0,0) \\ x_2 = 2 \rightarrow f(2) = \dfrac{8}{1} = 8 \rightarrow (2,8) \end{cases}$

$f'(x) > 0$ para $(-\infty, 0) \rightarrow$ Creciente $\Big] \rightarrow$ MAX

$f'(x) < 0$ para $(0, 2) \rightarrow$ Decreciente $\Big] \rightarrow$ MÍN

$f'(x) > 0$ para $(2, +\infty) \rightarrow$ Creciente

c)